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THE GEOMETRIC GRIDS OF THE HIERATIC NUMERAL SIGNS

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ABSTRACT

The paper discusses the geometrical designs of the hieratic numeral signs. It shows the regular-grid-patterns of squares upon which, the shapes of the already decoded hieratic numeral-signs, have been designed. Also, it shows the design of some hieratic numeral signs, based on subdividing the circle; and the hieratic signs of modular notation. It might reveal the basic geometrical level of understanding of anonymous ancient Egyptians who designed them some four thousand years ago.

KEYWORDS: Egyptian Mathematics, Rhind Papyrus, Moscow Papyrus, Hieratic Notation, Numbers, Modular Notation

1. INTRODUCTION

For almost fourteen decades, great efforts have been done by many scholars in order to interpret and introduce the legacy of the ancient Egyptian mathematics to the scientific community. The most known early publications were by, August Eisenloher (1877), Thomas Eric Peet (1923), and Arnold Buffum Chace *et al* (1927-1929), for interpreting part or all of the hieratic text of the so-called Rhind Mathematical Papyrus (RMP); and also by, W. W. Sturve (1930) for interpreting the hieratic text of the so-called Moscow Mathematical Papyrus (MMP); etc[1]. Thanks to the great efforts that have been done by many scholars in this area of knowledge; particularly their works pertaining to decoding the meanings of the hieratic numeral signs, which to some extent do not match their equivalents in the hieroglyphic writings. The hieroglyphic numeral system is as old as the days of King Narmer[2]; according to Archibaled (1930, 109-212) it goes back to 3500 BCE[3]. Gillings (1972, 4) commented on the hieroglyphic numeral signs of the ancient Egyptians, saying "the method used to represent numbers must have been at least easier than writing their phonetically equivalent words". Sixty two years before the publication of Gillings, Budge (1910, 127-128), based on the works of early scholars, showed the discovered way of writing numbers in Egyptian hieroglyphs by repeating and adding vertical strokes to represent numbers from 1 to 9, e.g., III means 3 and IIII means 4; and then from 10 upwards they used various signs[4] to represent the progressive hierarchy of numbers based on the power of ten, i.e., an arch ∩ denotes ten, a horn denotes 100, a lotus flower denotes 1000, a finger denotes 10,000, a frog denotes 100,000, and the ideogram of large denotes 1,000,000. That is, writing numbers in hieroglyphs was by adding strokes and figures[5]. In line with the opinion of Budge (1910, 4) concerning that hieratic writing is abbreviation of hieroglyphic writing, Georges Ifrah (1998, 171) said "hieratic signs were indeed derived from hieroglyphs"; however, he pointed out to that "the relationship between hieratic numerals and hieroglyphs is difficult to see". Most hieratic numeral signs seem follow different architectonic concept, which is not based only on the idea of adding characters of pre-assigned values to indicate numbers, i.e., the values of one and 10^n, where n equals any of the integers from 1 to 6 at most. Boyer and Merzbach (2010, Ch2) expressed their opinion concerning the transformation from the hieroglyphic numerals into hieratic numerals by saying "Numeration remained decimal, but the tedious repetitive principle of hieroglyphic numeration was replaced by the introduction of ciphers or special signs to represent digits and multiples of power of ten"; and added "the principle of cipherization introduced by the Egyptians some 4000 years ago, .. is one of the factors that marks our own system in use today". They meant, e.g., writing 26 using only two signs or writing 746 using only three signs. In contrast, similar to the opinion of Budge concerning the hieratic letters, Clagett (1991, 5) denoted the writing of the hieratic numerals by the word "abbreviated", which might imply writing in shorthand, but this is not the case in most hieratic numeral signs. From the artistic point of view, if one practiced the hieratic writing, one might notice that its basic-idea is based on drawing the outline of the profile of the hieroglyphic figure following either clockwise or anti-clockwise directions of motion, or both together, without closing the loop of the profile, particularly that for the live-beings in side-view, e.g., the anti-clockwise hieratic profile of the hawk (see the first black letter in RMPp50, in figure-5) looks like witting the two English letters *li*, in small. If one applied that on the hieroglyphic numerals, the results will in most numbers be different from what we read in the heretic manuscripts. That is, contrary to the case of the hieratic letters, most hieratic numerals are not abbreviations of the hieroglyphic numerals, e.g., the hieratic sign of number 7 is not abbreviation of its hieroglyphic sign . The author of this paper, therefore, investigated the question that "is there an ancient Egyptian unified concept leaded to the design of their hieratic numeral signs? As a result of reviewing the design typology of the hieratic numeral signs, the author of this paper suggests that hieratic notations are to many extent derived from specific geometrical system, which might support the note of Herodotus (440 BCE, Book II) on what might was the implication of the practice of measuring lands in Egypt by saying "from this practice I think Geometry first came to be known in Egypt" and might also imply that they knew the right–angled triangle (3, 4, 5) long before the days of Pythagoras[6].

2. THE NUMERIC GRID SYSTEM OF SQUARES

Based upon the discovered meanings of the hieratic signs of numerals by early scholars[7], and upon reviewing the design of each of these numeral signs, figure-1 shows that the hieratic shapes of integers (from 1 to 10) and the fractions 2/3, 1/2 and 1/3 were designed using the grid of a thematic rectangular, its dimensions are 4*3 units.

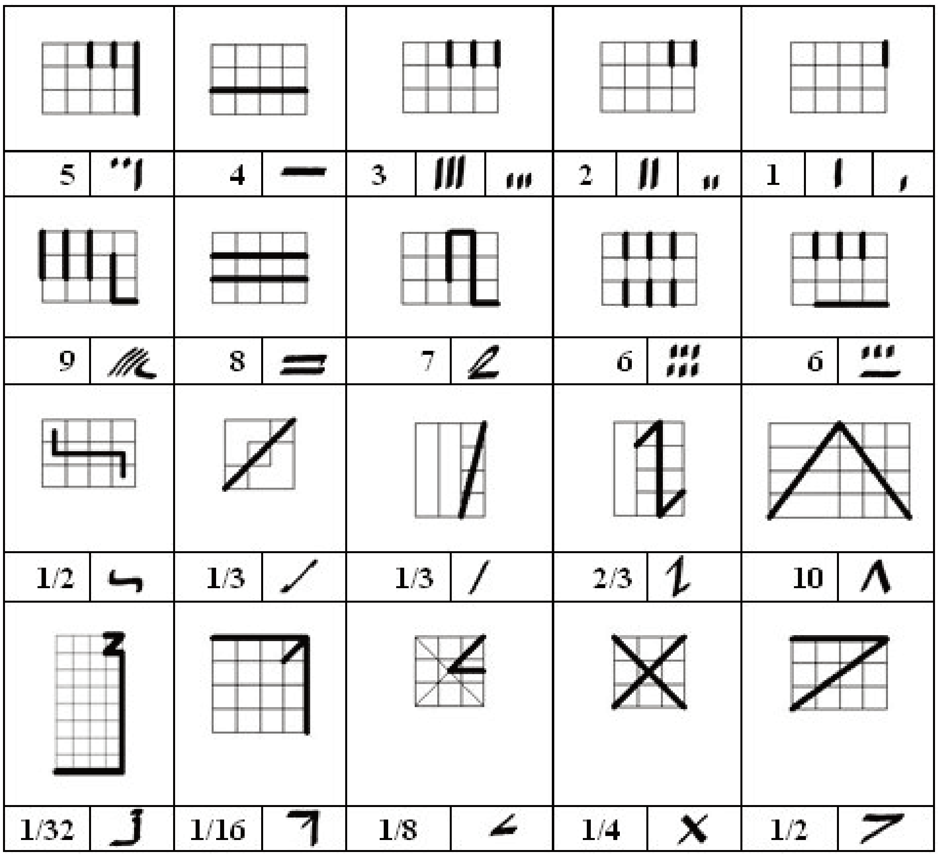

Figure-1. The design grids of the hieratic signs of integers from 1 to 10, and the fractions: 2/3, 1/2, 1/3, 1/4, 1/8, 1/16, and 1/32.

Designing the shapes of integers seem were based on that one is a side of any square–grid–unit of the thematic rectangle of 4*3 units, represented by thick lines. The hieratic sign of number 10 that looks like a cross section of a pyramid, and that has pointed and not arched crest, is not, therefore, imitating the corresponding hieroglyphic arched sign. More likely, it is a composite design from two diagonals of 5 units of two attached rectangles, which follows the same principle of representing a number by units but in inclined form. And this design idea of the sign of number 10 might imply that the designer knew well the right–angled triangle (3, 4, 5), as half the rectangular[9] that its dimensions is 4*3 units. On the other hand, the fractions 2/3, 1/2, and 1/3 were represented, in such a way, to mark the area deducted from, or remained in, that rectangle. The fraction 1/4 was represented by dividing any square into four quarters, e.g., a square of 3*3 units. The sign of the fraction 1/3, as it some times appears in the hieratic texts, e.g., in the "Egyptian Mathematical Leather Roll", as backslash tilted to 45° could be also the 3 square–units along the diagonal of the thematic square of 3*3 units. Similarly, the fraction 1/4 were also written, e.g., in RMPp59, as backslash tilted to 45° which represents the 4 units along the diagonal of a thematic square of 4*4 units. Also, in RMPp53-55, the fraction 1/2 was represented by a sign composed of 6 square-units from the thematic rectangular of 4*3 units that contains 12 square-units, namely: the four square–units in the middle row, the first square–unit, from the left side, in the upper row, and the first square-unit from the right side in the lower row.

In RMPp35, the binary fraction 1/16 was represented by a sign like a right angled corner of a square , where it marks the diagonal of one unit from a square of 4*4 units, and its upper and right sides. Similarly, the fraction 1/32 was represented by a sign, like the profile of a jaw that marks the upper and lower sides and the diagonal of one unit in the upper right corner of a rectangle of 8*4 units and its remaining right and bottom sides, i.e. 7 of 8 units, and 4 units, respectively. The fraction 1/8 was represented either by a sign that looks like the nowadays symbol of an angle , which marks one-eighths the area of a square of 3*3 units, or 4*4 units. In the decomposition of 2/37 in the 2/n table in RMP, the fraction 1/8 was written as two horizontal strokes of four units each and a dot of the whole one above them.

The thematic rectangular grid of 4*3 units allows also the generation of various forms of some integers, e.g., the numbers 4, 6, 7, and 9. The hieratic mathematical texts include other forms but all of them are based on the same grid system. For example, in MMPp10, number 9 was written to represent the sum of the parts (5+2+2) instead of writing it to represent the sum of the parts (2+2+2+3) ; number 7 was written in such a way that seems to represent (4 top +3 bottom) instead of writing it in the normal form as shown in figure-1; and number 4 was written as four separate ones instead of a horizontal line of four succeeding but linked units . In the decomposition of 2/9 in the 2/n table in RMP, number 6 was written as three horizontal strokes above each other and that each is composed of two units; in other problems of RMP it was written in two columns that each includes three horizontal strokes of one unit.

3. THE NUMERIC SYSTEM OF THE CIRCLE AND HORUS EYE

The design of the hieratic sign of the fraction 1/64 was found different. If it was unsymmetrical and its vertical stroke was shifted to the left, one may say it marks the bottom left square unit in a grid of 8*8 units, following the same deign concept of the other binary fractions. On the contrary, if it is symmetrical, which it is strongly the case, figure-2 shows possible justification that implies there might were additional geometrical information in the mind of its designer. It might was based on subdividing the circumference (or the area) of the circle P_1 into 64 parts where the radius r_1 of that circle is the vertical stroke that equal (10+1/8) times 1/64 of its circumference, and the horizontal stroke is the arc that equals twice of 1/64 of the circumference of an inner circle P_2 that its radius r_2 equals 9 times 1/64 of the circumference of the circle P_1.

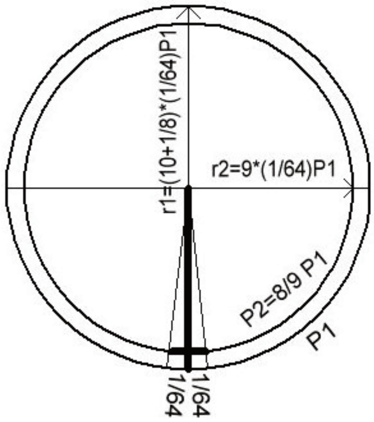

Figure 2. The probable geometric design of the hieratic symbol of 1/64

That is, the ratio between the circumference of P_2 and the circumference of P_1 is 8/9, i.e., $r_2/r_1 = 8/9$. Early scholars found no evidence in the ancient. Egyptian mathematical papyri that prove they subdivided the circle into any number of parts. However, on stone-walls, there is a perfect inscription that shows they subdivided the circle into 6 equal parts, as in the wheel of the chariot of Ramesses II in his temple in Luxor (Bongioanni 2004, 79). Also there is another example found in the ceiling of tomb 353 in Luxor (see Dorman, 1991) that shows they also subdivided the circle into 24 equal parts for astronomical purpose.

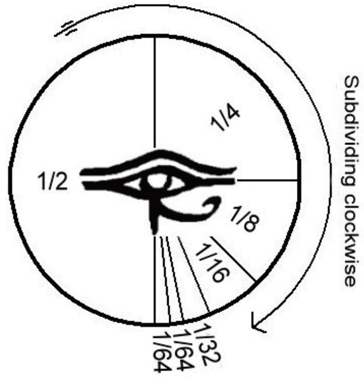

Figure 3. The binary clockwise subdividing of the circle and their likely artistic distribution sequence on the parts of the so-called eye of Horus; where 1/4 was assigned to the corona and 1/8 was assigned to the eye-lash.

As shown in figure-3, if the supposition concerning: they knew how to subdivide the circle into 64 parts was correct[10], it might was also a motive for inspiring the artistic-assigning of the binary fractions 1/2, 1/4, 1/8, 1/16, 1/32 and 1/64 to the 6 parts of the so-called the eye of Horus[11] ; that is, if we start from its left-side with 1/2 and rotate clockwise. In RMPp64, the binary fractions: 1/2, 1/4 and 1/8 were denoted by three signs that more likely are parts of the Horus eye. It seems that their selection and numerical meanings was based on drawing only the segments that are located in the upper left corner of the Horus eye. As known, the symbols imply 1/8, 1/2 and 1/4, from left to right respectively; and the signs of 1/4 and 1/8 are different from those that were derived from the regular thematic units, as was shown in section-2 above. It is also strange that the Horus eye sign for 1/2 is similar to the regular sign for 1/8 that was used in many geometric problems of RMP, e.g., RMPp53-54, and was shown in figure-1. That may imply that Horus eye fractions were used only in special mathematical context. Today in Egypt, the equivalent of the term "the eye" in Arabic is used to imply the property (money–in–kind) that could be subdivided into shares among its owners, e.g., a land, a building, a crop, etc. Horus eye might was used in the same way in Ancient Egypt, and that might had lead to its artistic subdivision to represent the binary fractions; and, that perhaps were used only in none geometrical problems, e.g., RMPp64.

4. THE COMPOSITE NUMERAL SYMBOLS

Furthermore, the design of hieratic signs for the numbers of more than one digit seem were based on the idea of composition. Some hieratic numeral signs were designed by including a multiplication factor of ten, which is imbedded in them in the form of either a dot or a stroke, as shown in figure-4. Examples of these signs of numbers below 1000 are found in the 2/n table in RMP. The dot of the power of ten was used in the sign of number 40 that denotes multiplying 10*4 units. That dot of ten is different from either the repeating dot like in the sign of number 30, denoting (10+10+10), or that which was used in some of the signs of hundreds, e.g. the sign of 300 that denotes (100+100+100). It is also different from the repeating inclined-stroke of one unit in the sign of 20 that denotes (10+10). Besides, either vertical stroke of three units, or horizontal stroke of two, or three, units were used to imply hidden multiplication factors of ten. The vertical stroke was used in the sign of number 50 to denote (10*(3+2)), and in the sign of number 70 to denote (10*(3+2+2)). The horizontal stroke was used in the sign of number 60 to denote (10*(2+2+2)); in the sign of number 80 to denote (10*(2+2+2+2)); and in the sign of number 90 to denote (10*(3+2+2+2)).

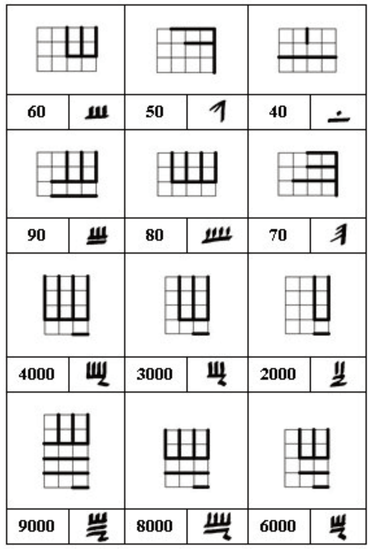

Figure 4. The design grids of some hieratic signs of integers, based on using either a stroke or a dot of the power of ten.

Contrarily, the hieratic sign of number 100 is not based on the same idea; it looks like a stretching form of its corresponding hieroglyphic sign of the horn. Also, the ancient Egyptian used combinations of vertical strokes of two units and horizontal strokes of one, two, or three units that each implies the power of ten, in order to compose the signs of thousands. For example, RMPp79 includes three signs of thousands: the sign of number 2000 denotes (10*10(10+10)), and the sign of number 3000 denotes (10*10(10+10+10)), and the sign of number 9000 denotes (90*10*10). Similar to the hieratic sign of number 100, the sign of number 1000, e.g., in RMPp52, is also not based on the idea of composition; it represents its corresponding hieroglyphic sign of the lotus flower in abstract form. Also, the sign of 10,000 that appeared in RMPp79, looks like the clockwise profile of its corresponding hieroglyphic sign of a bent finger.

5. THE MODULAR FORMS OF NUMBERS

There are also other forms of writing

numbers found in RMP, which might reveal that the ancient Egyptian had used early form of modular notation[12]. Its hieratic notation is similar to the idea of using the strokes and dots of ten. In RMPp48 that shows comparison[13] between reckoning the area of a circle its diameter equals 9 and the area of its circumscribed square, the numbers (16, 32, & 64) and (18, 36, 72, & 81) were not written using the usual Egyptian way, by putting the signs of the numbers of two digits (from 10 to 80) before the signs of the numbers of single digit. The hieratic signs of the numbers: 10, 30, 60, 70 and 80 were not used; instead, the ancient Egyptian had written them as 1, 3, 6, 7 and 8, respectively, implying how many times the module (modulo) ten is repeated in each of them, and put the remainders (6, 2, & 4) and (8, 6, 2, & 1), after them respectively, under an arc or a crescent that equates the dot of ten. The Ancient Egyptian modular notations seem different from the current idea of the so-called modular arithmetic notation[14] or "clock arithmetic" that is based on identifying one of the dividing points on the circumference of a circle that its perimeter equals the module in numeric term, where the position of each point represents set of numbers, e.g., today, 32 would be written in the cyclic or circular (mod 10) as 2 (mod 10) or $[2]_{10}$. In RMPp48, it seems that number 32 was written in linear or spread-out (mod 10) from right to left as three vertical strokes (of three units) that each represents ten, in addition to putting two separate strokes (of one unit) under the crescent that each represents one. It might be written today[15] in the form: (3*10) + 2 (mod 10) or 3+$[2]_{10}$, where 3 is the number of the linear or spread-out modules, 2 is the remainder, and 10 is the value of the module that the crescent represents. Also, in RMPp48, the modular dot of ten was used for writing the value of the diameter 9 of the circle, as part of the modular ten, denoting: 9 (mod 10) or $[9]_{10}$, where the diameter 9 was represented by the triple-symbol. On the other hand, in RMPp50, that its hieratic text is shown in figure-5, and that is based on RMPp48, the ancient Egyptian reckoned the area of a circle[16], its diameter equals 9, by finding the dimensions of the square that its area is equivalent to the area of that circle. The result of reckoning was 64, which was written as 60 and 4 under the modular crescent of ten denoting: 60 + 4 (mod 10) or 60+$[4]_{10}$. In RMPp48, number 64 was written similar to 32 that was shown above, using 6 strokes of ten instead of the sign of sixty, denoting (6*10) + 4 (mod 10).

Figure 5. Part from the so-called Rhind Mathematical Papyrus-RMP; problem 50 on reckoning the area of a circle that its diameter equals 9, by squaring 8/9 of that diameter; it is an example of the hieratic mathematical writings; used with the permission of the British Museum.

6. CONCLUSION

What have been discussed in this paper was an endeavor to show that the ancient Egyptians had designed most of their hieratic numeral signs (notations) based on using, e.g., the regular grid, of the thematic rectangular of 4*3 units, and its half right-angled triangle (3, 4, 5), which might indirectly imply they did know that triangle long before the days of Pythagoras. The paper showed as well that they used early form of modular notations in (mod 10), which seems were based on linear or spread-out representation and not based on identifying the sets of numbers along the circumference of a circle that its perimeter represents the module (modulo). Besides, the design-concept of the hieratic numerals/notations that are presented in this paper was integrated and complete incremental-development in numerical writings that also allows diversity in unity. Perhaps, it was designed at specific point in time with certain degree of flexibility to generate other forms for numbers over time, using the same grid.

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Special thanks to Dr. Jeffrey Spencer, the "Deputy Keeper of Ancient Egypt & Sudan" at the British Museum in London, for sending me part of the facsimile of Rhind Mathematical Papyrus, and giving me the permission to use it in this paper.

NOTES

[1] The early publications on the subject are many; David Eugene Smith (1924, 557-559) in his review of T. E. Peet's translation in the Bulletin of American Mathematical Society, mentioned that professor Archibald had listed large number of articles upon the subject, written by some seventy-five different scholars; see also the bibliography of Archibald's article (1930, 109-121) in Science Magazine.

[2] See the oldest form of hieroglyphic numerals on the royal mace of Hierakonpolis (King Narmer's mace) in the book of Clagett (1989, 751).

[3] Since Hieroglyphs was used in Egypt from the days of Narmer (or Min) who was the first king in the Egyptian records (see, Herodotus 440 BCE, Book II), the story of Plato (330 BCE) in the dialogue "Phaedrus" about inventing the writing in Egypt by Theuth during the reign of King Thamos, might was for inventing the hieratic technique for writing on papyrus.

[4] Budge (1910, 88-93) said the sign of 100 is cord/rope and did not give meaning to the sign of 10; in my opinion their meanings still are used in the Egyptian Arabic language of today; that is, the sign of 10 looks like an arch, which is pronounced as *Aqud* in Arabic and means also a decade of 10 years, and the sign of 100 looks like a horn, which is pronounced as *Qurn* in Arabic and means also a century of 100 years; and large is implied by the two hands in the ideogram.

[5] E.g., the number 102114 would be written in hieroglyphs from right to left in this way .

[6] Many scholars have rejected the supposition that the ancient Egyptians might know the right–angled triangle (3, 4, 5) long before the days of Pythagoras. For example, T. L. Heath (1962, 352) wrote "there seems to be no evidence that they knew the triangle (3, 4, 5) is right–angled; indeed, according to the latest authority (T. Eric Peet, the Rhind Mathematical Papyrus, 1923), nothing in Egyptian mathematics suggests that Egyptians were acquainted with this or any special cases of the Pythagorean theorem". Corinna Rossi (2004, 71) wrote, "On the case of the 3–4–5 triangle, we do not possess any explicit early mathematical sources recording its knowledge." Annette Imhausen (2009, 901) wrote, "While it cannot be excluded that Egyptian mathematics and architecture might have used Pythagoras triplets, most notably 3–4–5, it must be kept in mind that our actual evidence for this is only on measurements of the remains of buildings, which — as we have seen — may will be misleading."

[7] See the various forms of the hieratic numeral signs in the book of Gillings (1982, 255-259),

and in the book of Clagett (1991, 314-325) that were taken from G. Moller.

8 All the hieratic signs/words included in this article, excluding figure-5 of the so-called Rhind Mathematical papyrus, are the handwriting of its author, and the direction of hieratic words/letters are from right to left.

9 Archibald (1930, 109-121) mentioned problem 6 in Golenishchev papyrus (Moscow mathematical papyrus), which concerns finding the sides of a rectangular enclosure of 12 units area and ratio of sides as 1:(1/2+1/4), and the lengths of the sides are found 4 and 3; accordingly, if the ancient Egyptian was able to measure the four sides of a polygon and reckoned its area, what could hinder him from measuring its two diagonals and numerically relate their equal values to its sides?

10 In modern degrees, the corresponding lengths of circular arcs are: 180°, 90°, 45°, (22+1/2)°, (10+1/4)°, and two of (5+1/2+1/8)°, respectively.

11 Gillings (1982, 211) and Clagett (1991, 319) showed the distribution of the binary fractions on the Horus eye parts; and, Annette Imhausen (2009, 790) tried to refute the opinions concerning representing the binary fractions by parts of the Horus eye, denoting it as a "myth"; since there are no hard evidence proves this distribution. In this regard, hard evidences that are recorded in artifacts have no numerical meanings; e.g., an inscription without any text saying it is the Hours eye in both directions (left and right eyes)  is found in the tomb of Sennefer in Luxor (Bongioanni, 2004, 187). The left Horus eye has astronomical reference, where it is an imitation of the lunar–eye at the night of the full moon that is recorded in the ceiling of Denderah temple (Planche-19, Description de l'-Egypte 1820, 38) as well as in Dendera Zodiac, denoting the gray tone in the circle of light of our moon  (Aboulfotouh, 2007, 23-37).

12 The hieratic word for module, modulus, or modulo  might be pronounced either as *Pot or Phot;* for example, it is appeared in the fourth and last line in RMPp50, as shown in figure-5.

13 See Archibald (1930, 109-121).

14 See for example the idea of cyclic or clock arithmetic in the book of Ash *et al* (2010, 31-33).

15 It might also mean in modern modular arithmetic notation: summing [0 (mode 10) + 2 (mod 10)].

16 See Gillings (1982, 139-140), and in this regard, given that the area of the ancient Egyptian circle that its diameter equals 9, was reckoned as 64, which is more than the area of a similar circle when reckoned, using the modern π of 3.141592…; accordingly, 64 might represent the area of a quasi-circle composed of four parabolic curves that are constructed on the sides of its inner square that its diagonal is equal to the diameter 9.

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