Retrieving the Design Method of the Islamic Decagonal Girih Patterns

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Abstract.

Peter J. Lu, *et al*, in their recent report in Science magazine, suggest that the decagonal girih patterns on the Darb-i Imam shrine, Isfahan, Iran, 1453G, are quasi-periodic and were constructed by tessellation, using a set of five tile types, which they called girih tiles. They doubted the architectonic ability of the medieval designers and accordingly they devaluated both the processes and the outputs of their creativity. Contrary to what they have suggested, this paper shows the manual and swift technique to design and implement the decagonal patterns, based on drawing three types of overlapped but hidden grids of equal intervals (two of them are tilted on both sides by 72° and 36°), using only a T-square and two triangles: 18°/72° and 54°/36°. Besides, it shows a simple method that the students of the 1st and 2nd years of architecture can use in order to draw any decagonal pattern found in Egypt or other countries; as well as to create new patterns using repeated modular-units, and to retrieve the authentic motifs and visual-identity in new local architectures.

Key words: Architecture Identity, Architecture Education, Islamic Motifs, Decagonal Patterns.

1-Introduction.

This paper discusses a technical point on the micro architectonic level, i.e., the decagonal Islamic patterns, but it primarily aims to retrieve the authentic architecture identity in the cities of countries of North Africa and the Near East. It links the competition of arts in the architectonic realm with the diversity of ideologies in the era of globalization. Today, Architecture identity has become in the forefront of scientific debates. In one hand, the scientific communities of developed countries dedicate the current decline of some Arab countries in most areas of scientific, technological, architectural and cultural aspects to applying imperfect developmental ideologies. The international journal "Nature" had published a series of biased articles on this subject (Nature news), which enraged many intellectuals from different Near Eastern countries. Some of them have responded with comparing between the conditions of the current developed countries and the less developed countries in the Near East and

North Africa during the medieval ears, i.e., the golden eras of the Islamic empire. However, it was without discussing what have been changed, e.g., what are the causes of development boom of the Capitalist nations and the sharp decline in sciences and economies of the developing Arab nations? On the contrary, there are researchers and intellectuals who do not support the illogical notions. They compare the results of the nowadays discoveries, as well as the architectural and cultural development with what have been done in the Islamic empire during the Medieval eras. The paper of Peter J. Lu et al, on the Islamic Girih patterns is a good example (Peter J. Lu et al, 2007); however, as will be shown hereafter, their claims warp the medieval architectonic design process and lessen the value of the design-patents of the local architects who produced them during the Medieval golden ears. Despite they did not declare clearly that the advancement during the medieval period was done by the Muslim scholars as have been stated by other scholars, e.g., in the fields of astronomy (Nallino 1911) and architecture (Castéra 1999). In the realm of architecture, it shows that the creative ideas by local medieval architects (Muslims and non-Muslims) were the outcomes of, or have been inspired or controlled by, the Islamic thoughts. There is no hard evidence that support any claim about that similar architecture products were designed before then. Besides, this situation may imply that despite the current anti-backwardness campaign, the nonbiased western researchers respect the achievements of the Islamic civilization.

However, enforcing the new trends of the so-called globalization on Muslim world might threaten the continuity of interpreting the local identities in their architecture, i.e., the physical character of cities in North Africa and the Near East**.** This issue concerns primarily the transformation of the architectonic-urban dialogues, which are usually supported by the political regimes. In Egypt, at the beginning of the 19^{th} century, Mohamed Ali started encouraged an architectural movement in order to make Cairo's visual image similar to the European cities, and was continued by his successors (see e.g., El-Rafey, 1930). In the 20^{th} century, the transformation was influenced by establishing the systematic architectural study in Cairo University, which introduced the internationally new architectural trends into the local architectural education. Accordingly, it consolidated the industrial sector of the western countries more than retrieving and emphasizing the authentic architectural trends (Aboulfotouh 2005b) that Mohamed Ali's regime had started to abolish its use hundred years earlier, and was accented by Khedive Ismaeil at the late 19^{th} century. Now, at the beginning of the 21^{st} century, the transformation is combined with replacing the basic static and abstract shapes (forms) of the 20^{th} century architecture, by the dynamic forms. That by entering into the realm of applying the concepts of the geometric topology (see, e.g., Borisovich, *et al.* 1985) on the architectural transformations, by moving, rotating, compressing, or stretching the static shapes or forms

The pace of this kind of movement seems was faster than the process of understanding the basic scientific knowledge behind creating the medieval and authentic architectural Islamic or Coptic images. Apparently, from the point of view of the young architects, following the new architectural trends may seem easier than investigating the process of creating the architectural designs during the golden Islamic eras. Besides, today one can hardly find books in the Arabic library that might were written by the architects of the medieval eras or earlier, and that show precisely their design philosophy and the process of creating the architectural Islamic motifs. It seems that the architects of those days (and perhaps the case is the same in other cultures too) had never written these kinds of books like those we read it in the other social sciences. Most of what we have now are postulations by some foregone or local researchers, who did not read and understand the books written by the famous medieval scholars and historians; hence, their opinions are highly doubtful, particularly their approaches on interpreting the Islamic ideology into geometrical patterns, but the foregone scholars produce high quality books though. These books (e.g., Castéra 1999 & Yves 1997) benefit the students of architecture in the western counties, because they can read the foreign texts, and many of the Arab students can hardly read and understand these texts, and many of them review only the drawing figures. Concerning the Islamic patterns, the Muslim scholars on the contrary, search only on analyzing the final outputs of these patterns without showing the process of designing and developing them. Al-Nahass's book is an example (Al-Nahass 2007); despite it includes many patterns, it lacks the explanation. This condition shows one of the arrays of weaknesses in the realm of authenticating the architectural education in the Arab and Muslim counties.

In the Egyptian realm, it is simply the lack of the authentic references for educating our architectural students, and re-continuing what Mohamed Ali had abolished, nearly two hundred years ago. The lack of architectural books from the golden Islamic eras might imply that the architectural education in these periods was handed from generation to generation on verbal and practice bases, without any written documents. Despite, their progress and creativity in architecture designs, it is hard to find any sign that proves there was systematic architectural education in these golden eras. Accordingly, in this context, the point in time when the political regime started to support any new architectural trend is the same moment of stopping the continuity of handing the verbal architectural courses from generation to generation. That is, this process of verbal and practical education did not include studying the philosophy and the history of architecture that started with the time of establishing the department of architecture in Cairo University. In the early decades of the 20th century, the intruded foreign architectural trends had dominated the Egyptian cities and weakened the competitive ability of the authentic architectural/spatial dialogues of the Islamic trends. The works of the local but foreign architects did support that discontinuity. In addition, even within the context of the systematic architectural studies, the authentic architectural trends were never given hands and feet on legitimate, constituent and communal bases, due to the lack of political will (Aboulfotouh 2005b).

The Egyptian case is nearly similar to the cases of other Muslim countries. In these countries, the current transformations of their architectonic-urban dialogues are indirectly dominated by the new trends of the western countries, leaving no chance for those of the Arab and Islamist world to conserve their architectural identities. Many young architects from the Muslim counties imitate the designs of others without understanding the basic sciences of either the most authentic images or the newest innovative images. As an example, regarding the new topological trends, very few architects understand what topology is, due to that topology (e.g., Borisovich, *et al* 1985) is not taught as basic architectonic subject in the first year of our architecture education.

In this paper, we show the development of an architectonic thought until it reached its climax of application and before its decline due to the above-mentioned reasons that abolished the authentic architectural image in great parts of our cities. The hidden message of this paper is to show that the architects of these days can easily master the architectonic dialogue of our cities as it was in the golden eras, and making it able to compete with the modern architectural trends, using the application of the architectural Islamic patterns as an example. However, its core objective lies within the domain of the architectural education, by retrieving the easiest process for designing and creating the architectural patterns that conserve and retrieve the authentic identities of the Islamist cities, but also fits the current needs, the available resources and the human skills. We focus here on the decagonal girih patterns that are found in Churches and Mosques of Medieval Egypt and elsewhere.

2- Numbers and basic shapes.

Mathematics and descriptive geometry are essential subjects in the architectonic field. They are as old as the architectonic profession, which go back to the days of the pyramids builders, nearly 5000 years ago. In those days, the architect was not only the designer and the producer of the architectonic enclosures; he was more like a philosopher who understands the metaphysical laws of nature (Aboulfotouh 2007b). Thus, his designs were not only to create the most suitable forms and spaces that enclose the human functions, and that realizes safety, flexibility, privacy, territoriality, and protection, but also to create the spatial image that emphasizes the identity of his society in the three dimensional representations. The design language he used draws its letters from the field of mathematics. In this field, the architect studies the numbers and their geometrical representations. Al-Maqrizi, an Egyptian Judge and the known historian of the 15^{th} century, said in his famous book *Al-Khetat* "the priests of ancient Egypt had thought about the relation between the basic geometrical shapes (circle, square, triangle, etc.) and the motions of planets and the moon" (Al-Maqrizi 1849). The Physagoreans observed each of these shapes as it indicates a natural number, e.g., the triangle is 3, the square is 4, the pentagon is 5, the hexagon is 6, the decagon is 10; besides, the dote is one and the line is 2 (McLeish 1992). Perhaps, one may follow the Physagorean approach on representing the numbers in geometrical ways until the shape becomes close to a circle, which might depict the numbers from 360 until infinity. These basic shapes (or basic systems in the plane of two dimensions) are the primary elements for constructing any space, starting from the man-made spaces of three dimensions (on earth) up to the God-made spaces of eleven dimensions, like the architectonic structures of the celestial systems, i.e., the complex systems (Ahoulfotouh 2004).

In this paper, we focus on the use of the basic shapes as the thematic modules for creating the architectonic patterns, either in straight or curved planes, but not in the stretched or compressed topological planes. Since the corners of all basic and concentric shapes are distributed on equal arc distances on the perimeter of a circle, we propose two conditions for defining the master architectonic shape(s): each of them is constructed by or from triangles, and the total sum of its inner angles should be 360°. Despite that the triangle of equal sides and each of its three angles equals 60° (equi-angular), is the first basic shape, the second condition discards it out of the proposed definition of the master or the very basic architectonic shape, but it makes the right angle triangle, the basic repeating unit, or the module of constructing any basic architectonic shape.

Accordingly, the square is the first basic architectonic shape that qualifies the two conditions of the above definition. Without using the compass, we cannot draw a square using only a T-square and a scale; we need an additional drawing tool. Perhaps, the ancient architectonicians had thought about that and thus they invented the triangular tool of the right angle that could be used to draw any basic shape. Using simple calculations, one can retrieve their way of thinking, in order to produce the triangle for each basic shape. For the square, it seems they had divided 360° by 4 and they did got 90°, which is not below 90°, thus they divided it again by 2 to get 45°. Accordingly, they invented the triangle 45°/45°, in order to construct a grid of squares, without using the compass, and using a scale to measure only one side. Similarly, for constructing the hexagon and the hexagonal grid, they invented the triangle 30°/60°, by dividing 360°/6, it gives 60°, and then dividing 60°/2 gives 30°. For the pentagon, if we divide 360° by 5, we get 72; and then if we divide 72° by 2, we get 36°, which do not complement 72°. Hence, for the pentagon we use two triangles: 18°/72° and 36°/54°. Today one can hardly find these last two triangles in the stationery shop, but one can find the triangle of various angles instead, and that we can adjust it to any tilt to get diverse pairs of two complemented angles.

Similarly, we can get the values of the two angles of the triangle that we might use to draw any basic shape and construct its grid. Taking into consideration that the needed triangle(s) for drawing a square is the same as for drawing the octagon and the needed triangle for the pentagon is the same as for the decagon. Using this very simple rule, one can identify the types of triangles that are needed in order to draw the grids for constructing any architectonic pattern that is based on, includes, or be made of the repeated concentric shapes.

Moreover, the repetition of basic shapes in/on a plane surface in order to construct a grid is conditional to the order of symmetry of that shape. Symmetry in basic shapes could be defined as the geometric characteristic that enables one to divide that shape into mirrored parts around an imaginary axis. The symmetrical order of each basic shape, therefore, depends on how many symmetrical axes it has, which should be more than two axes. The square has four axes, the pentagon has five axes, and the hexagon has six axes. However, this self-architectonic characteristic do not imply that the basic shape could be used alone as a uniformly repeated modular unit in order to construct a grid in/on a plane surface. In addition to the triangle of equal sides, the only two basic shapes that have this second characteristic are the square and the hexagon, and that might was the reason why the triangles 45°/45° and 30°/60° were and still are the most important triangular-tools in architectonic design. If we draw the triangular grid for the hexagons with the aid of the triangle 30/60, as shown in fig-1 (left-side), we shall notice that this grid repeats vertically with, or on, equal intervals (like x) and with 1.15 x on the horizontal direction, creating the repeating square unit ($30x$ * $30x$). It implies that the grid of squares is the master grid of their basic shapes. Not only this, due to that the rest of the basic shapes do not have the characteristic of uniformly repeating them on the plane surface, the grid of squares is their master grid too, being the base for constructing the repeated module of all basic shapes. This is the reason for asserting earlier that the total sum of the inner angels of the master architectonic shape(s) should be 360°, making the other basic shapes as secondary subdivisions inside this master basic shape.

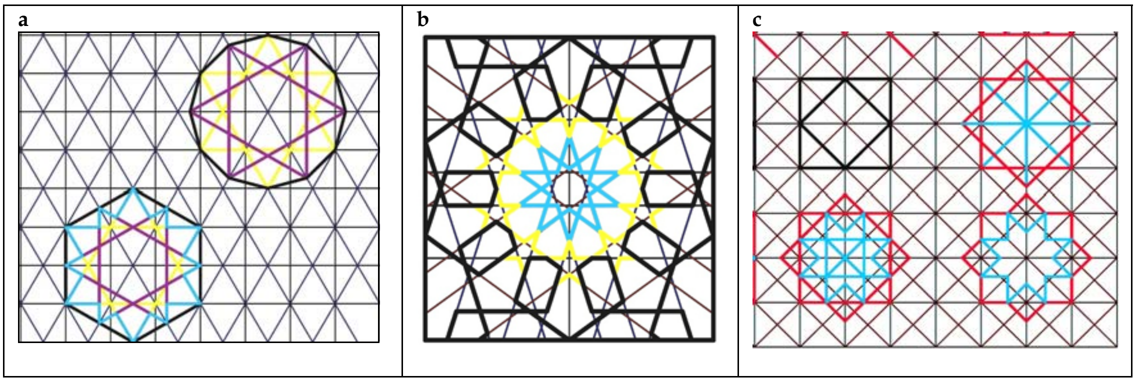

Figure-1: The grids of patterns of different basic shapes: (*a*) triangle/hexagon, (*b*) pentagon /decagon, and (*c*) square/octagon.

3- Interpreting ideology into geometrical patterns.

The Ideology of any nation implies their thoughts-on, believes-in, and interpretations-of three laws in their daily life and working mechanisms: the metaphysical law of creation, the law of adoration, human relations and justice; and the law of governance and development (Ahoulfotouh 2005b). In most ancient civilizations, their architects were given the authority to study and to interpret the metaphysical laws of nature in their architectonic designs. They were not forbidden to imitate and depict the shapes of any living or mythological creatures, as pictures on walls and ceilings or as statues inside or outside their buildings. Due to the myths that were created by imposters and bad soothsayers on some of this artistic figures and artifacts (that were perfectly grafted with the architecture design) the illiterate people worshiped them, and forget that Allah (the divine name of God, with his multiple names in different tongues) is the sole creator and the first cause of all living forms (Aboulfotouh, 2006). Worshiping the fabricated statues and art-figures were insulted and prohibited by many profits (as mentioned in Quran) and by the philosophers of the ancient civilizations. They showed the difference between respecting the works and the achievements of man in the figure, or the hidden natural characteristic of any creature, and that were made by the sole creator; and they had emphasized on worshiping only

God, the great Geometer. The history says that not before the establishment of the strong Islamic governance system, in the Arabian-Peninsula and expanding its administrative jurisdiction to include other countries that formed, in latter periods, the geographic domain of the Islamic empire, any other ideological regime was not able to abolish completely that illiterate ways of adorations. Only with the empowerment of the verses of the holy Quran it was finally accomplished with no return during the early decades of the Muslim empire, in its expanding geographic domain, and under the umbrella of the Islamic governance. This holy objective was accomplished in two ways, but their corresponding administrative actions differed from country to country. First, prohibiting the imitation of the living or mythological creatures in any kind of art or grafting it with any architecture. Second, the existing artifacts that include these kinds of things, but were not worshiped in illiterate way, may remain on conditional bases, based on the opinion of the Islamic governor "*Wally*", at that time. Al-Maqrizi said, "Amr Ibn Alass, the first Muslim governor of Egypt, did not demolish the Ancient Egyptian temples and statues based on knowing that the Egyptians do not worship them. Similarly, in other period, Ibn Tulun demolished a sundial statue in ancient Heliopolis, not because of any illiterate worship practices, but it was to prove the opposite of a fictitious story about it" (Al-Maqrizi 1849).

Concerning the first condition, in those days the architects of the Islamic empire, whether they were Muslims or none Muslims, were enforced to find other alternatives to beautify their new buildings and that do not violate the verses of the Holy Quran. Their applications differed from place to place depending on their local and inherited architectonic experiences. All of them did find the solution in using the basic shapes to interpret the metaphysical laws of the great geometer on the walls, the facades, the domes and/or the ceilings of their buildings (Hossam Aboulfotouh 2006). Besides, Islam did not prohibit the studies of astronomical and astrological subjects, but on the contrary, during the golden eras of the Muslim empire, some advancement in these fields were done by many Muslim scholars (Nallino 1911 & Pedersen 1993), continuing the researches of the ancient Egyptian priests and the Greek philosophers, and that were supported by studying mathematics and geometry.

The scientific heritage in these two fields contains many theories that were used in the architecture designs. For example, interpreting the dynamic motions of the celestial bodies, that occur in multiple frames and multiple dimensions into two-dimensional geometrical forms on a plane surface were initiated by the ancient Egyptians, in the so-called the *Ankh* diagram, the symbol of cosmic life (Aboulfotouh 2007b). When observing the motion of any planet, the number of times a planet changes its shape, position, or appearance can be interpreted in a basic form. For example, during the lunar month, one can observe the eight different shapes of the moon, from the first crescent until it becomes full moon and then reduces until it disappears to start the crescent of the new lunar month. The ancient Egyptians thus related the octagonal shape to the moon and the lunar year. In addition, they related the square to the Sun for its four seasons. Similarly, they related the triangle to Venus, the square also to Mars, and the hexagon to Saturn (Al-Maqrizi 1849). In short, they related the basic shapes to the cosmic motion that each indicates, i.e., for them, basic shapes were the interpretations of these celestial motions.

Moreover, many ancient civilizations respected number 10, perhaps, because it equals the sum of the first four natural numbers 1, 2, 3 and 4, and it is the divider of the most ancient and basic astronomical measurement units: meter, kilometer or above that (Aboulfotouh 2005a). The Physagoreans, and Euclid, talked about the characteristics of the perfect numbers such as 6, 28, & 496 and proposed their definition without a cosmic reference. In previous study, Aboulfotouh investigated the architectonic and physical characteristics of the perfect numbers in the three-dimensional space (Aboulfotouh 2004). Using the characteristics of natural numbers and their indications from diverse ideological points of view (that differ from place to place and from civilization to civilization), architects did use in their designs the numbers and their corresponding basic shapes that symbolize the ideologies of their societies. In the early eras of Islam, in Egypt, the Christian architect who won the competition of designing *Ibn Tulun* Mosque in Cairo (Al-Maqurizi 1849) did use different hexagonal patterns in more than hundred windows in this mosque. The decagonal patterns are closely linked with the *Shiaa* Doctrine (*el-Ashria* or the ten Imams), thus we find these patterns on most *Shiaa* buildings, like the *Mehrab* of Al-Azhar mosque in Cairo and Darb-i Imam Shrine in Asfahan, and in Fas, as shown in fig-2. In addition, the Egyptian Christians respected the number ten, and used the decagonal shapes in churches in Egypt during the 7^{th} centuries, e.g., in Hanging church in Cairo. The *Sonnies'* Muslims did use the octagonal patterns that are associated to the moon and the lunar year (*Hijri* calendar). Similarly, one can use the symbolic meaning of any number in order to create any architectural pattern that speaks on behalf of the society that respect or understand this geometrical pattern.

	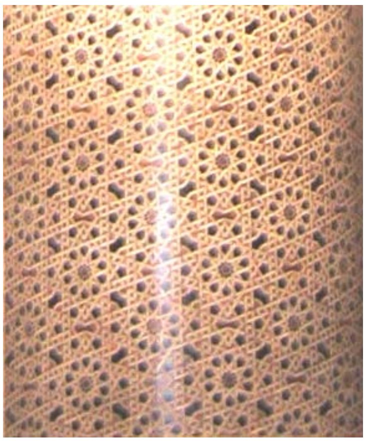	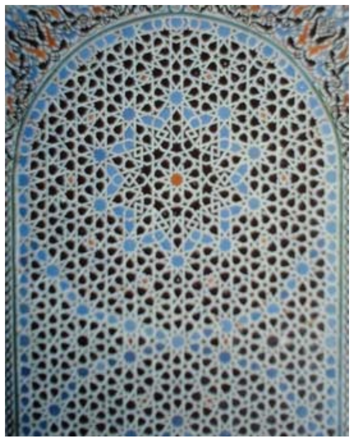
Darb-i Imam shrine, *Asfahan, Iran*	*Mehrab of Al Azhar Mosque,* *Cairo, Egypt*	*Attarine Madrasa (school)* *Fas, Morocco*

Figure- 2: Examples of using the decagonal patterns in Muslim *Shiaa* buildings.

4- Drawing and designing the decagonal patterns.

Figure-3 shows the postulation of Peter J. Lu *et al*, in their reports as they suggest that the decagonal girih patterns on the Darb-i Imam shrine are quasi-periodic and were constructed by tessellation, using a set of five tile types. (Peter J. Lu et al, 2007a).

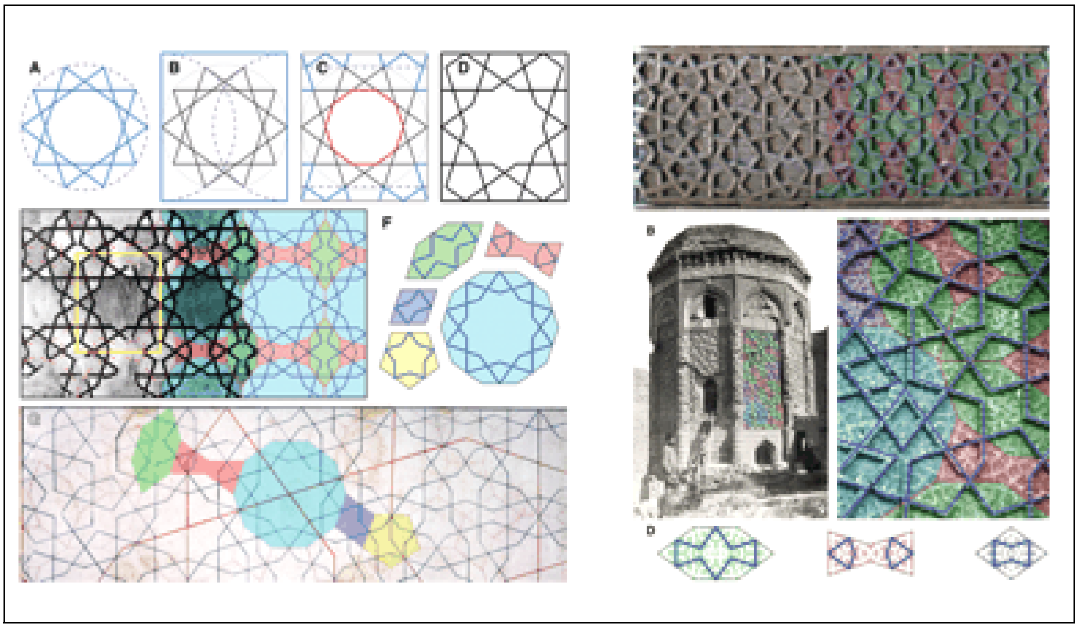

Figure-3: Peter J Lu's proposed girih tiles in the left-side, and on Gunbad-I Kabud tomb tower, in Maragha-Iran in the right-side (figure source: Peter J. Lu *et al*, 2007a).

Responding to this claim, Aboulfotouh's e-letter to Science magazine showed the hidden grid approach for designing and drawing the decagonal girih patterns (Aboulfotouh 2007a), which will be explained hereunder in details. See also the response of Peter J. Lu *et al* to Aboulfotouh Science magazine; they thought that the inner-level of details in an intermediate part of the decagonal patterns that show only the hexagons and had been used on the Gunbad-I Kabud tomb tower, in Maragha-Iran, include curves which is not true, see fig-2 (Peter J. Lu *et al* 2007b).

Contrary to the approaches of the mathematicians towards understanding the process of designing and implementing these patterns, architects can easily design them manually; using only a scale, a T-square, and various types of triangles, and produce their working drawings in 1:1 scale for the artisans. Since these patterns do not include circular curves, we can draw them swiftly without using the slow compass. For the decagonal stars, we can use two triangles instead: 18°/72° and 36°/54°, which are not produced today, but the triangle of various angles is an alternative.

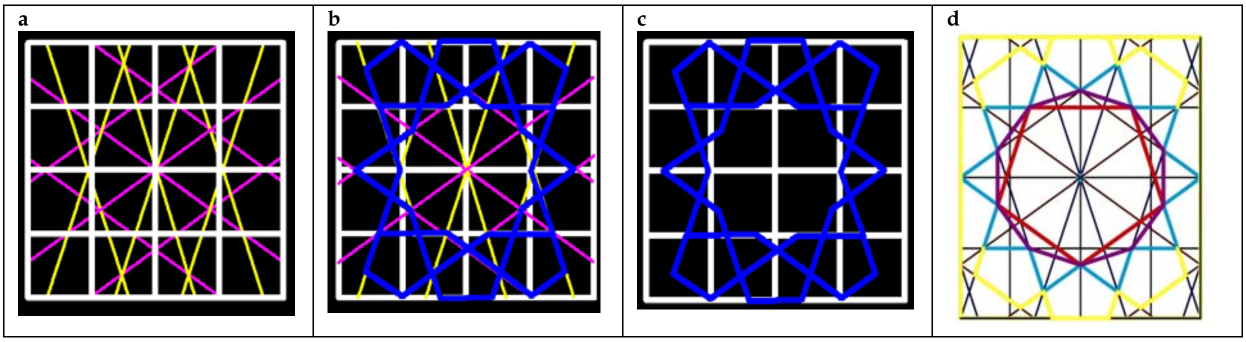

Figure-4: The process of drawing decagonal stares without using the compass (from *a* to *d*).

To draw a decagonal-star, as shown in fig-4, if x is the radius of its inner circle that the ten sides of the inner decagon are its tangents, we draw a square that its side equals $4x$, and draw the perpendicular grid $4x*4x$ inside it. Using the triangle 18°/72°, and starting from the "center of the square", we draw the second but inclined grid every x, and its tilt equals 72° on both sides. Similarly, using the triangle 36°/54°, we draw the third but inclined grid, every x, and its tilt equals 36° on both sides. Then, we can observe the perimeters of the decagonal-star and its surroundings, i.e., one of the multiple design outputs of the three hidden-grids.

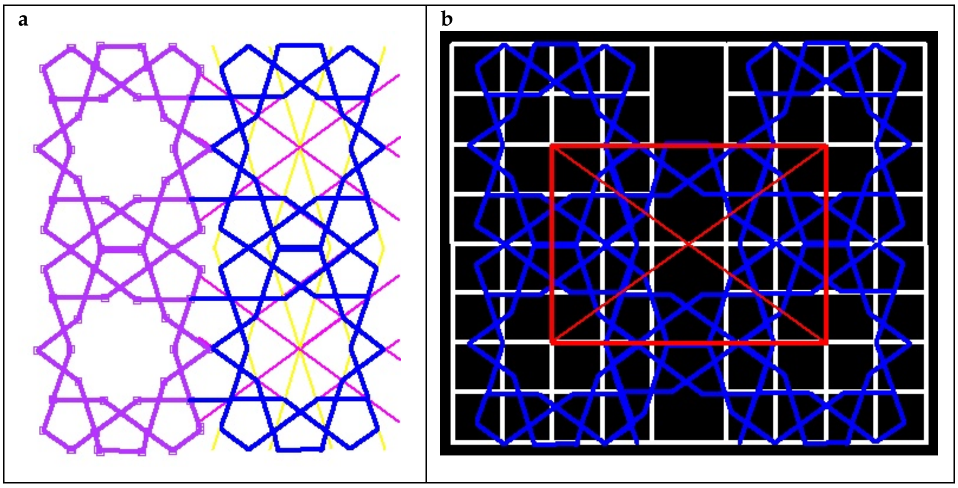

Figure-5: The decagonal patterns: (*a*) follows the rhythm $1.4x$, $2x$, and (*b*) follows the rhythm $2x$, $1.5x$, $2x$ with the repeated thematic-unit of $4x*5.5x$ (in red color).

If we repeated the $4x*4x$ unit vertically, we would produce a vertical strip of repeated stars, every $4x$. On the horizontal direction, there are various options. While the horizontal grid repeats every $4x$, the vertical grid may follow various continuous rhythms, e.g., $2x$, $1.5x$, $2x$ or $0.7x$, $2x$, $0.7x$. The rhythm $2x$, $1.5x$, $2x$ generates a fifth decagonal-star in the middle of each four stars, creating the stagger-shape, as shown in fig-5 (right-side), within the repeated thematic-unit of $4x*5.5x$. The rhythm $1.4x$, $2x$ generates the grid-shape, as shown in fig-5 (lift-side) and the repeated thematic-unit of $4x*3.4x$.

If one analyzed the girih patterns that were designed by the architects in different places and during different periods, one may observe that some of them are small portions of repeated thematic-units that its vertical but hidden grid-sides were rotated, inclined, and/or located outside the domains of the design-motifs. In other cases, like that on the Drab-i Imam Shrine in Asfahan, the inner-grids were subdivided in order to design the second and third level of inner-details.

5- Conclusion.

Great architectures are the outputs (and outcomes) of understanding the basic knowledge and the accumulation of sciences of diverse and interrelated architectonic disciplines. Despite it seems that the architects of the golden Islamic eras did not write any books on their philosophy of design, their masterpieces that still exist until now show and prove that they were mastering these kinds of basic knowledge and that made them able to produce their great architecture. Architecture education in our universities should give emphasis to the basic sciences that would support the design abilities of our students and enhance their innovative skills. This is not only to cope with the new trends but also to retrieve the old authentic lessons and to understand how to do it in the likely effective and contemporary ways, in order to conserve the identity of our societies. Regarding the example of decagonal patterns that we discussed in this paper, designing and implementing these patterns without tessellation were and still are not difficult tasks. This is contrary to the opinions of the scholars who might try to limit the authentic architectonic abilities in this marvelous realm that we call the man-made architecture and that was inspired by the architecture of God.

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